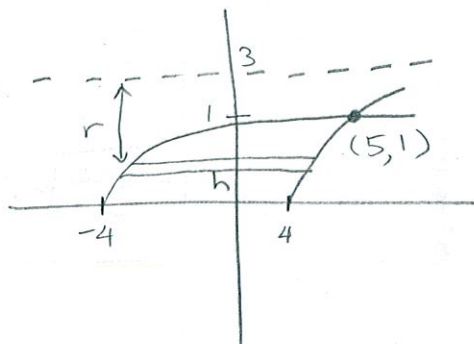


The region bounded by $y = \frac{1}{3}\sqrt{x+4}$, $y = \sqrt{x-4}$ and $y = 0$ is revolved around the line $y = 3$.

SCORE: ____ / 12 PTS

Find the volume of the resulting solid.



$$\frac{1}{3}\sqrt{x+4} = \sqrt{x-4}$$

$$\frac{1}{9}(x+4) = x-4$$

$$x+4 = 9x-36$$

$$40 = 8x$$

$$x = 5$$

$$y = \frac{1}{3}\sqrt{x+4}$$

$$3y = \sqrt{x+4}$$

$$x = 9y^2 - 4$$

$$y = \sqrt{x-4}$$

$$x = y^2 + 4$$

$$\begin{aligned} & \textcircled{2} \int_0^1 \textcircled{1} 2\pi (3-y) (y^2+4 - (9y^2-4)) dy \\ &= 2\pi \int_0^1 (3-y)(8-8y^2) dy \\ &= 2\pi \int_0^1 (24-8y-24y^2+8y^3) dy \\ &= 2\pi (24y-4y^2-8y^3+2y^4) \Big|_0^1 \\ &= 2\pi (14) \textcircled{4} \\ &= \textcircled{1} 28\pi \end{aligned}$$

DEFEND YOUR WORK
IF YOU USED DISK/WASHER
METHOD (MUCH MORE
COMPLICATED)

The base of a solid is the region in the xy - plane bounded by $y = f(x)$ and $y = g(x)$.

SCORE: ____ / 3 PTS

If cross sections perpendicular to the x - axis are equilateral triangles, the volume of the solid is 12 .

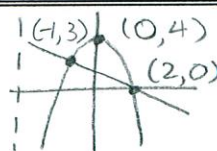
Find the volume if cross sections perpendicular to the x - axis are semicircles.

$$\frac{\sqrt{3}}{4} \int_a^b s^2 dx = 12 \quad \textcircled{1}$$

$$\int_a^b s^2 dx = \frac{48}{\sqrt{3}} = 16\sqrt{3}$$

$$\frac{\pi}{8} \int_a^b s^2 dx = 2\pi\sqrt{3} \quad \textcircled{1} \quad \textcircled{1}$$

Consider the region bounded by $y = 4 - x^2$ and $y = 2 - x$.



SCORE: ____ / 12 PTS

- [a] If the region is revolved around the line $x = -4$, write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid

$$4 - x^2 = 2 - x$$

$$0 = x^2 - x - 2$$

$$0 = (x - 2)(x + 1)$$

$$x = 2, -1$$

$$y = 4 - x^2$$

$$x = \pm \sqrt{4 - y}$$

$$y = 2 - x$$

$$x = 2 - y$$

THE ANSWERS FOR [i] AND [ii]

☆ $(-\frac{1}{2})$ POINT EACH TIME YOU FORGOT

dx or dy

- [i] using the disk or washer method (**NOTE: You do NOT need to simplify your integrand.**)

$$\textcircled{1} \int_0^3 \pi \left((\sqrt{4-y} + 4)^2 - (2-y+4)^2 \right) dy \quad \textcircled{2}$$

$$\textcircled{1} + \int_3^4 \pi \left((\sqrt{4-y} + 4)^2 - (-\sqrt{4-y} + 4)^2 \right) dy \quad \textcircled{\frac{1}{2}} \quad \textcircled{2}$$

☆ (-2) POINTS

IF YOU

SWITCHED

THE ANSWERS

FOR [i] AND [ii]

☆ $(-\frac{1}{2})$ POINT EACH

TIME YOU FORGOT

- [ii] using the shell method (**NOTE: You do NOT need to simplify your integrand.**)

$$\textcircled{\frac{1}{2}} \int_{-1}^2 2\pi (x+4) (4 - x^2 - (2-x)) dx \quad \textcircled{\frac{1}{2}} \quad \textcircled{2}$$

- [b] Suppose the region is the base of a solid.

Cross sections perpendicular to the x -axis are isosceles right triangles with the hypotenuse in the base region. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.



$$\textcircled{\frac{1}{2}} \int_{-1}^2 \frac{1}{4} (4 - x^2 - (2-x))^2 dx \quad \textcircled{1} \quad \textcircled{1}$$

Find the area between the curves $y = 4x^2 - 7$ and $y = x^2 - 4x$ over the interval $-1 \leq x \leq 2$.

SCORE: ____ / 6 PTS

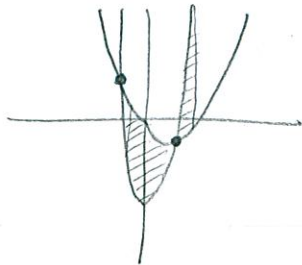
NOTE: The answer is NOT 6.

$$4x^2 - 7 = x^2 - 4x$$

$$3x^2 + 4x - 7 = 0$$

$$(x-1)(3x+7) = 0$$

$$x = 1, -\frac{7}{3}$$



$$\begin{aligned} & \left(\frac{15}{2}\right) \int_{-1}^1 (x^2 - 4x - (4x^2 - 7)) dx + \int_1^2 (4x^2 - 7 - (x^2 - 4x)) dx \\ &= \int_{-1}^1 (-3x^2 - 4x + 7) dx + \int_1^2 (3x^2 + 4x - 7) dx \\ &= \left(-x^3 - 2x^2 + 7x\right) \Big|_{-1}^1 + \left(x^3 + 2x^2 - 7x\right) \Big|_1^2 \\ &= (4 - -8) + (2 - -4) \\ &= 18 \end{aligned}$$